An Unintuitive Game

Here is a video walkthrough of the solutions.

Sumer challenges Sohum to the following game. In this game, there is a maximizing player and a minimizing player. Both players take turns adding numbers to the end of the sequence. The maximizing player wants to maximize the **last** number in the sequence, and the minimizing player wants to minimize it.

On a player's turn, they take the previous number in the sequence and create the next number by either:

- floor dividing it by 2
- multiplying it by 3 and adding 1

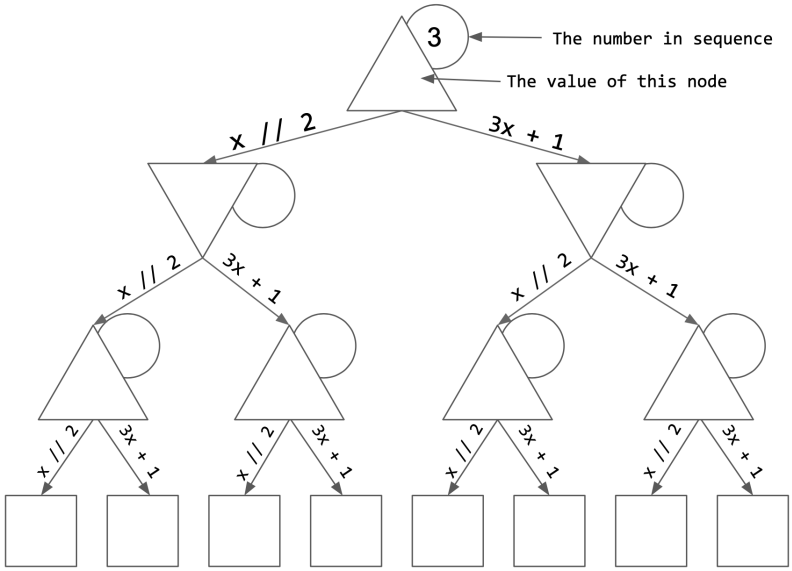
The sequence starts with 3, and can only contain numbers in the range 1 - 5. If a number goes out of bounds after an operation, it wraps around. For example, floor dividing 1 by 2 is zero, which wraps around to 5. Finally, the maximizing player will always start and get two turns. The minimizing player will get one turn, giving there a total of three turns.

Here is an example of a sequence.

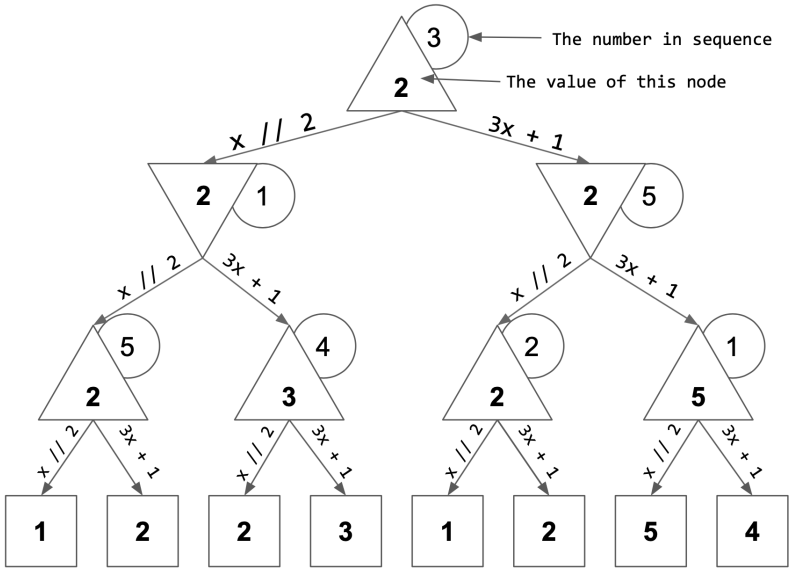
1. We start with **3**.
2. The maximizer chooses to multiply 3 by 3 and add 1, giving us 10, which wraps around to **5**.
3. The minimizer chooses to floor divide 5 by 2 to get **2**.
4. The maximizer chooses to multiply 2 by 3 and add 1, giving us 7, which wraps around to **2**.
5. The last number in the sequence is ***2***.

(a) Fill in the minimax game tree for the following game. Typically in game trees only store the **value** of each node, but for this game it will be helpful to keep track of the current number in the sequence, so an additional circle has been given for that.

Note that no circle is written besides the leaf nodes because the value of each leaf node is the current number! Also note that we wrote 3 in the first circle to get you started (since the sequence starts with 3).



Solution:

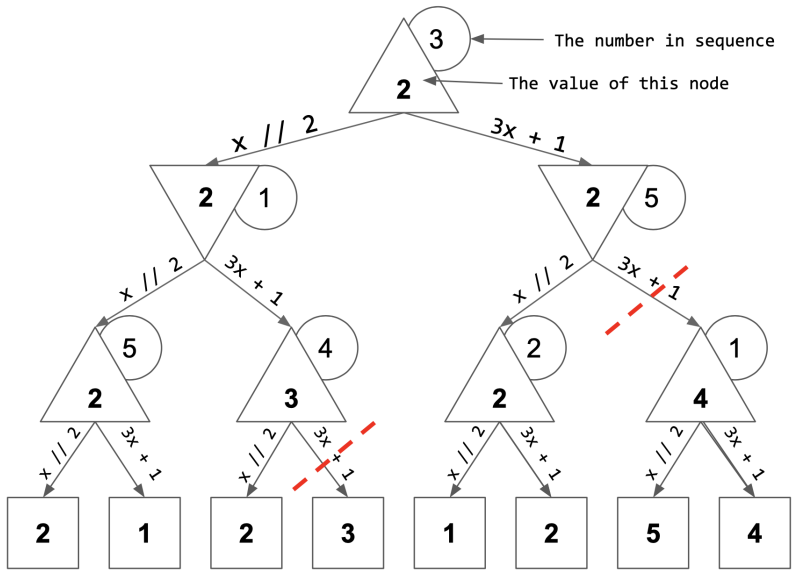


(b) Assuming both players play optimally, what is the last number in the sequence?

Solution: 2

(c) Using the game tree from the part a, which branches can be pruned with alpha beta pruning? Cross out the branches, if any, in the previous tree.

Solution:



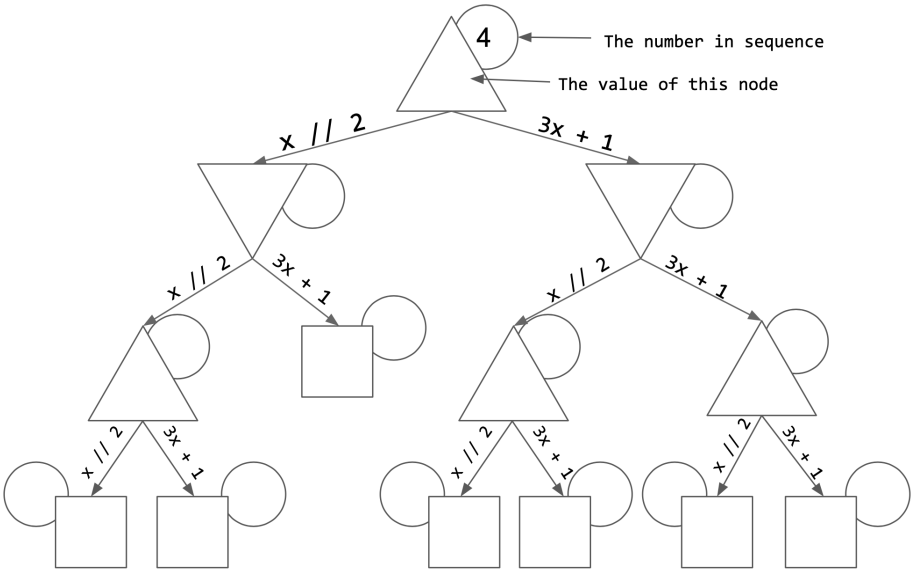
Explanation: The right branch under the maximizer to 3 can be pruned because we know the minimizer can always take its left child to get a value of 2. The maximizer has one child with value 2, so it must be ≥ 2, and can never be better than the minimizer's left child.

After examining the left branch, we know the root must have value at least 2. Examining the right minimizer's first child, we see that it has value 2, so the right minimizer has value ≤ 2, and can never be better than the root's left child.

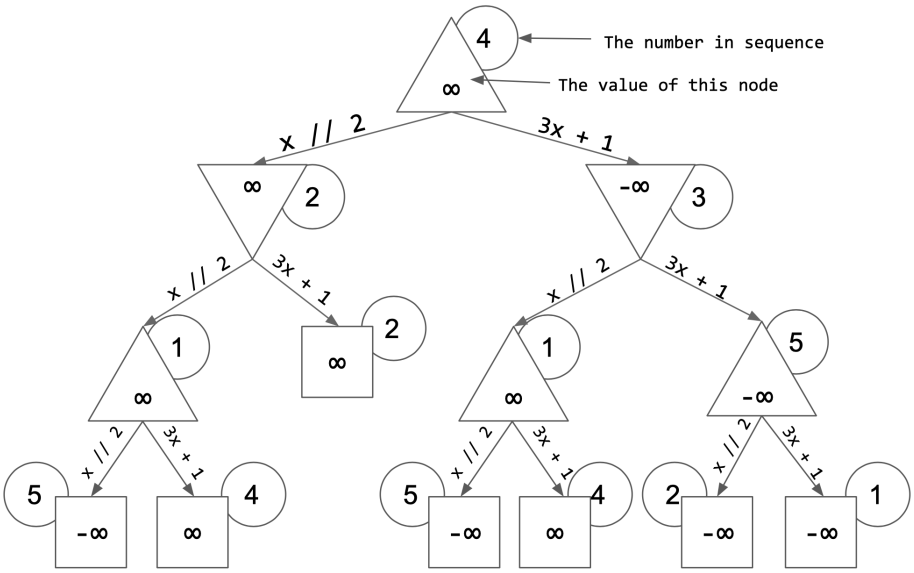
(d) Now, suppose that we keep the game *exactly* the same but we change two things:

1. We start with 4.
2. The maximizer **wins** if there is a duplicate in the sequence and the minimizer wins if the numbers are unique.

Fill in the game tree for the modified game. Note that a circle is written beside each leaf node because the value of each leaf node isn't the current number! Hint: Use ∞ and $-\infty$ to represent the maximizer winning and the minimizer winning, respectively.



Solution:

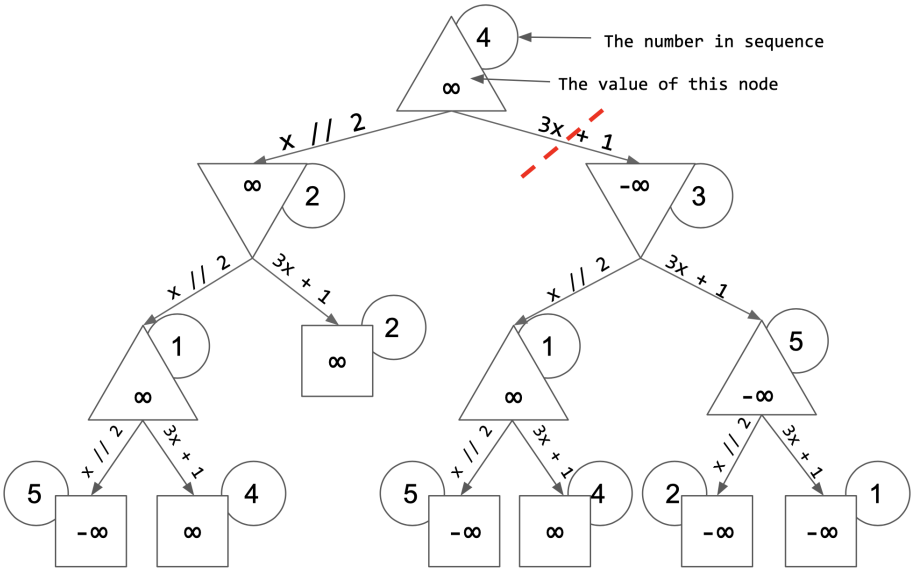


(e) Assuming both players play optimally in this modified game, who wins?

Solution: The maximizer!

(f) Using the game tree from part c, which branches can be pruned with alpha beta pruning? Cross out the branches, if any, in the previous tree.

Solution:



Explanation: After examining the root's left child, we see that the maximizer already has a winning option, so it doesn't matter what the right child of the root is.