

Prime Factors

[Here is a video walkthrough of the solutions.](#)

Determine the best and worst case runtime of `prime_factors` in $\Theta(\cdot)$ notation as a function of N .

```
1  int prime_factors(int N) {
2      int factor = 2;
3      int count = 0;
4      while (factor * factor <= N) {
5          while (N % factor == 0) {
6              System.out.println(factor);
7              count += 1;
8              N = N / factor;
9          }
10         factor += 1;
11     }
12     return count;
13 }
```

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

Solution:

Best Case: $\Theta(\log(N))$, Worst Case: $\Theta(\sqrt{N})$

Explanation: In the best case, N is some power of 2. Then the inner while loop will halve N each time until it becomes 1. At this point, both the inner and outer while loop conditions will be false and the function will return. Halving N each time results in a $\Theta(\log N)$ runtime.

In the worst case, N will not be divisible by any value of `factor`. This means we increment `factor` by 1 each time until `factor * factor > N`. This is at most \sqrt{N} loops.