Heap Mystery

Here is a video walkthrough of the solutions.

We are given the following array representing a min-heap where each letter represents a **unique** number. Assume the root of the min-heap is at index zero, i.e. `A` is the root. Note that there is **no** significance of the alphabetical ordering, i.e. just because B precedes C in the alphabet, we do not know if B is less than or greater than C.

Array: [A, B, C, D, E, F, G]

Four unknown operations are then executed on the min-heap. An operation is either a `removeMin` or an `insert`. The resulting state of the min-heap is shown below.

Array: [A, E, B, D, X, F, G]

(a) Determine the operations executed and their appropriate order. The first operation has already been filled in for you!

1. `removeMin()`
2. ______________________________
3. ______________________________
4. ______________________________

Solution:

1. `removeMin()`
2. `insert(X)`
3. `removeMin()`
4. `insert(A)`

Explanation: We know immediately that A was removed. Then, after looking at the final state of the min-heap, we see that C was removed. Then, for A to remain in the min-heap, we see that A must have been inserted afterwards. And, after seeing a new value X in the min-heap, we see that X must have been inserted as well. We just need to determine the relative ordering of the `insert(X)` in between the operations `removeMin()` and `insert(A)`, and we see that the `insert(X)` must go before both.

(b) Fill in the following comparisons with either $>$, $<$, or ? if unknown. We recommend considering which elements were compared to reach the final array.

1. X ______ D
2. X ______ C
3. B ______ C
4. G ______ X

Solution:

1. X ? D
2. X > C
3. B > C
4. G < X

Reasoning:

1. X is never compared to D
2. X must be greater than C since C is removed after X's insertion.
3. B must also be greater than C otherwise the second call to `removeMin` would have removed B
4. X must be greater than G so that it can be "promoted" to the top after the removal of C. It needs to be promoted to the top to land in its new position.