Fill in the Blanks

Here is a video walkthrough of the solutions.

Fill in the following blanks related to min-heaps. Let `N` is the number of elements in the min-heap. For the entirety of this question, assume the elements in the min-heap are **distinct**.

1. `removeMin` has a best case runtime of ________________ and a worst case runtime of ________________.
2. `insert` has a best case runtime of ________________ and a worst case runtime of ________________.
3. A ________________ or ________________ traversal on a min-heap *may* output the elements in sorted order. Assume there are at least 3 elements in the min-heap.
4. The fourth smallest element in a min-heap with 1000 elements can appear in ________________ places in the heap.
5. Given a min-heap with $2^N - 1$ distinct elements, for an element
   - to be on the second level it must be less than ________________ element(s) and greater than ________________ element(s).
   - to be on the bottommost level it must be less than ________________ element(s) and greater than ________________ element(s).

   Hint: A complete binary tree (with a full last-level) has $2^N - 1$ elements, with N being the number of levels.

Solution:

1. `removeMin` has a best case runtime of $\Theta(1)$ and a worst case runtime of $\Theta(logN)$.
2. `insert` has a best case runtime of $\Theta(1)$ and a worst case runtime of $\Theta(logN)$.
3. A **pre order** or **level order** traversal on a min-heap can output the elements in sorted order.
   Explanation: The smallest item of a min heap is at the top, so whatever traversal we choose must output the top element first in a complete binary tree. Only preorder and level-order have this property.
4. The fourth smallest element in a min-heap with 1000 distinct elements can appear in **14** places in the heap.
   Explanation: The 4th smallest item can be on the 2nd, 3rd, or 4th level of the heap.
5. Given a min-heap with $2^N - 1$ distinct elements, for an element -
   - to be on the second level it must be less than $2^{(N-1)} - 2$ element(s) and greater than **1** element(s).

- to be on the bottommost level it must be less than **0** element(s) and greater than N **- 1** element(s). **(must be greater than the elements on its branch)**

Explanation: An element on the second level must be larger than the root and less than the elements in its subtree. There are $2^{(N-1)} - 2$ elements in the subtree of an element on the second level: half the elements in the tree minus the root, then subtracting off the node itself.

An element on the bottom level must be greater than all elements on the path from itself to the root. A min heap with $2^N - 1$ elements has N levels, so there are N - 1 items above it on a path to the root.