Graph Conceptuals

Here is a video walkthrough of the solutions for all parts but part e. Here is a video walkthrough for part e.

Answer the following questions as either **True** or **False** and provide a brief explanation:

1. If a graph with n vertices has $n-1$ edges, it **must** be a tree.

   False. A tree **must** be connected.

2. The adjacency matrix representation is **typically** better than the adjacency list representation when the graph is very connected.

   True. The adjacency matrix representation is usually worse than the adjacency list representation with regards to space, scanning a vertex's neighbors, and full graph scans. However, when the graph is very connected, the adjacency matrix representation has roughly same asymptotic runtime in these operations, while "winning" in operations like hasEdge.

3. Every edge is looked at exactly twice in **every** iteration of DFS on a connected, undirected graph.

   True. The two vertices the edge is connecting will look at that edge when it's their turn.

4. In BFS, let $d(v)$ be the minimum number of edges between a vertex v and the start vertex. For any two vertices u, v in the fringe, $|d(u) - d(v)|$ is **always less than** 2.

   True. Suppose this was not the case. Then, we could have a vertex 2 edges away and a vertex 4 edges away in the fringe at the same time. But, the only way to have a vertex 4 edges away is if a vertex 3 edges away was removed from the fringe. We see this could never occur because the vertex 2 edges away would be removed before the vertex 3 edges away!

5. Given a fully connected, directed graph (a directed edge exists between every pair of vertices), a topological sort can never exist.

   False. Consider the graph constructed as follows: for all vertices i, j such that $i < j$, draw a directed edge from i to j. A valid topological ordering of this graph is simply enumerating the vertices: $1, 2, 3, \ldots .N$.