Multiple MSTs

Recall a graph can have multiple MSTs if there are multiple spanning trees of minimum weight.

(a) For each subpart below, select the correct option and justify your answer. If you select "never" or "always," provide a short explanation. If you select "sometimes", provide two graphs that fulfill the given properties — one with multiple MSTs and one without. Assume G is an undirected, connected graph.

1. If **none** the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

2. If **some** of the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

3. If **all** of the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

(b) Suppose we have a connected, undirected graph G with N vertices and N edges, where all the **edge weights are identical**. Find the maximum and minimum number of MSTs in G and explain your reasoning.

Minimum: _________
Maximum: _________

Justification:

(c) It is possible that Prim's and Kruskal's find **different** MSTs on the same graph `G` (as an added exercise, construct a graph where this is the case!). Given any graph `G` with integer edge weights, modify G to **ensure** that Prim's and Kruskal's will always find the same MST. You may not modify Prim's or Kruskal's.
Hint: Look at subpart 1 of part a.