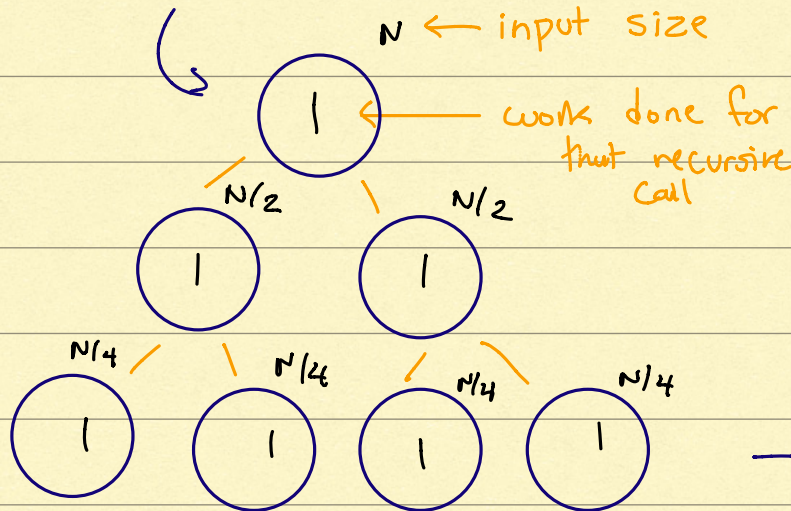


# GUIDE TO ASYMPTOTICS OF TREE RECURSION

## STEPS

1. Draw out the recursive tree

→ a node for every function call



void f(int n) {

if (n > 0) {

g(n); //  $g(n) \in \Theta(1)$

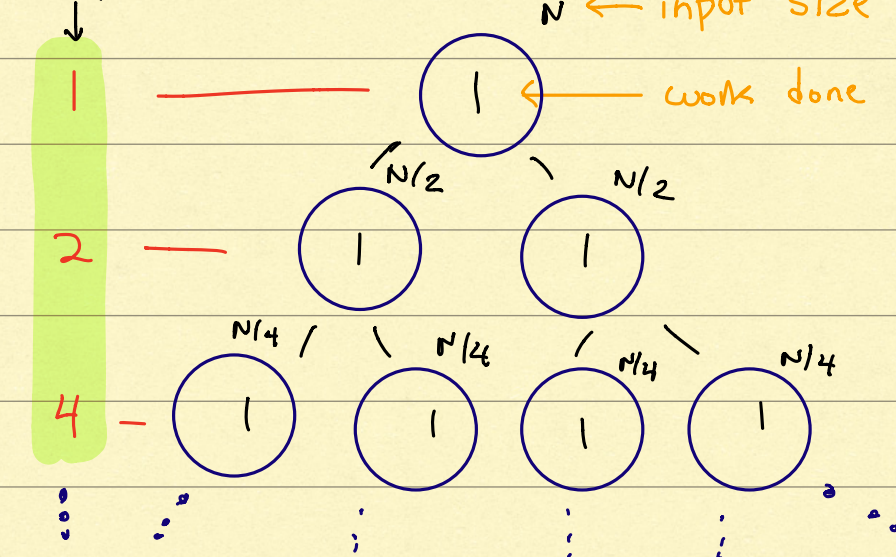
f(n/2);

f(n/2);

→ usually 3 levels is enough!

2. Determine work per level (using recursive tree from above)

work per level



3. Recognize which sum we are dealing with:

→ "arithmetic" sum:  $1 + 2 + 3 + 4 + \dots + N \sim N^2$

→ "dominating" sum :  $1 + 2 + 4 + 8 + \dots + N \sim N$

→ "constant" sum :  $\underbrace{N + N + N \dots + N}_N \sim N^2$

↳ (nearly always needed) calculate height of tree

→ First, notice that our sum is  $1 + 2 + 4 + \dots$  which matches the dominating sum shown above. All we need is the last term.

→ Notice further that each level does  $2^{\text{level}}$  work, where level 0 is when the input size is  $N$  and the levels count down.

→ Finally, we see that the last term in this sequence is  $2^H$ , where  $H$  is the height of the tree, or  $\log_2 N$

4. Output final answer!

We get  $2^{\log_2 N} \rightarrow \Theta(N)$