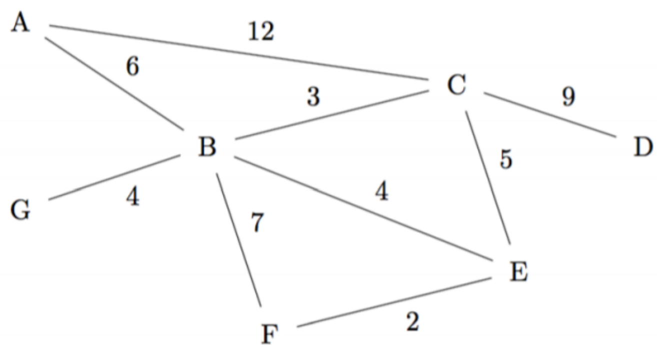
Introduction to MSTs

Here is a video walkthrough of all parts of this problem.



(a) For the graph above, list the edges in the order they're added to the MST by Kruskal's and Prim's algorithm. Assume Prim's algorithm starts at vertex A. Assume ties are broken in alphabetical order. Denote each edge as a pair of vertices (e.g. AB is the edge from A to B)

Prim's algorithm order:
Kruskal's algorithm order:

Prim's algorithm order: AB, BC, BE, EF, BG, CD
Kruskal's algorithm order: EF, BC, BE, BG, AB, CD

(b) Is there any vertex for which the shortest paths tree from that vertex is the same as your Prim MST? If there are multiple viable vertices, list all.

Vertex B, A, or G

(c) True/False: Adding 1 to the smallest edge of a graph G with unique edge weights must change the total weight of its MST

True, either this smallest edge (now with weight +1) is included, or this smallest edge is not included and some larger edge takes its place since there was no other edge of equal weight. Either way, the total weight increases.

(d) True/False: The shortest path from vertex A to vertex B in a graph G is the same as the shortest path from A to B using only edges in T, where T is the MST of G.

False, consider vertices C and E in the graph above

(e) True/False: Given any cut, the maximum-weight crossing edge is in the maximum spanning tree.

True, we can use the cut-property proof as seen in class, but replace "smallest" with "largest".